Journal of Structural Geology, Vol. 7, No. 1, pp. 115 to 117, 1985
Printed in Great Britain.

0191-8141/85 $03.00 + 0.00

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The use of the orientation tensor for the description and statistical testing of fabrics

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(*Received* 9 *February* 1984; *accepted in revised form* 20 *June* 1984)

Abstract—A measure of the degree of preferred orientation of directional data defined by Woodcock as the ratio of the maximum and minimum eigenvalues of the orientation tensor (S_1/S_3) has a disadvantage when used as the statistic for a test of uniformity. This drawback arises because the distribution of S_1/S_3 under random sampling is not independent of the shape or K value of the sample. An alternative strength parameter also based on the eigenvalues, is proposed.

INTRODUCTION

THE QUANTITATIVE description of fabric strength is vital in an increasing number of studies which attempt to explore the relationship between fabric development and other microstructural characteristics of tectonites. In a recent review of methods of fabric description, Woodcock & Naylor (1983) advocate the orientation tensor of Scheidegger (1965) for three-dimensional orientation data. As these authors point out the relative magnitude of the three normalized eigenvalues (S_i) of the orientation tensor offers a convenient way of classifying simple fabrics spanning the range between unimodal clusters and great-circle girdles.

To express the intensity or strength of such fabrics the ratio of the maximum to minimum eigenvalues (S_1/S_3) has been proposed (Woodcock 1977, Woodcock & Naylor 1983). This note points out two disadvantages of

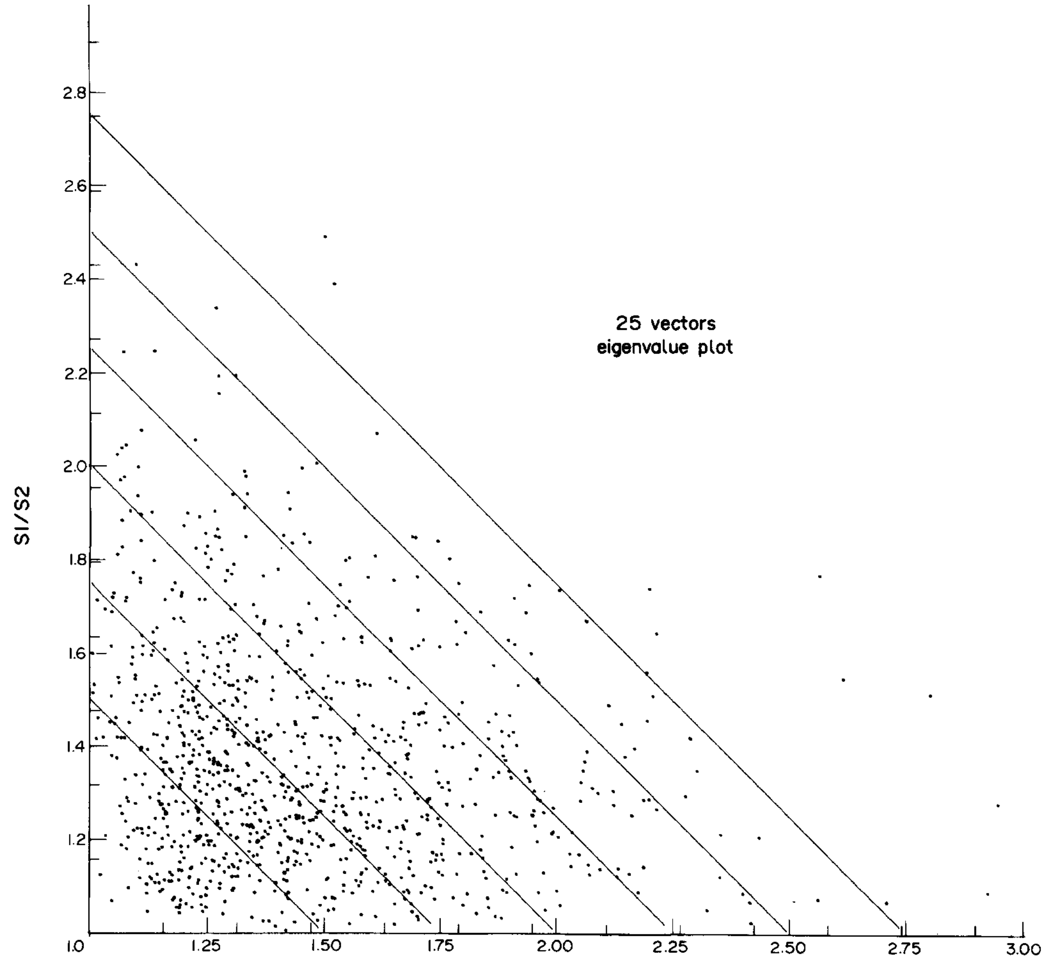

Fig. 1. Eigenvalue ratio plot showing the orientation tensor calculated for 1000 random samples (each of 25 vectors) taken from a uniform distribution of directions. The distribution of samples is compared to lines of constant S_1/S_3 ratio.

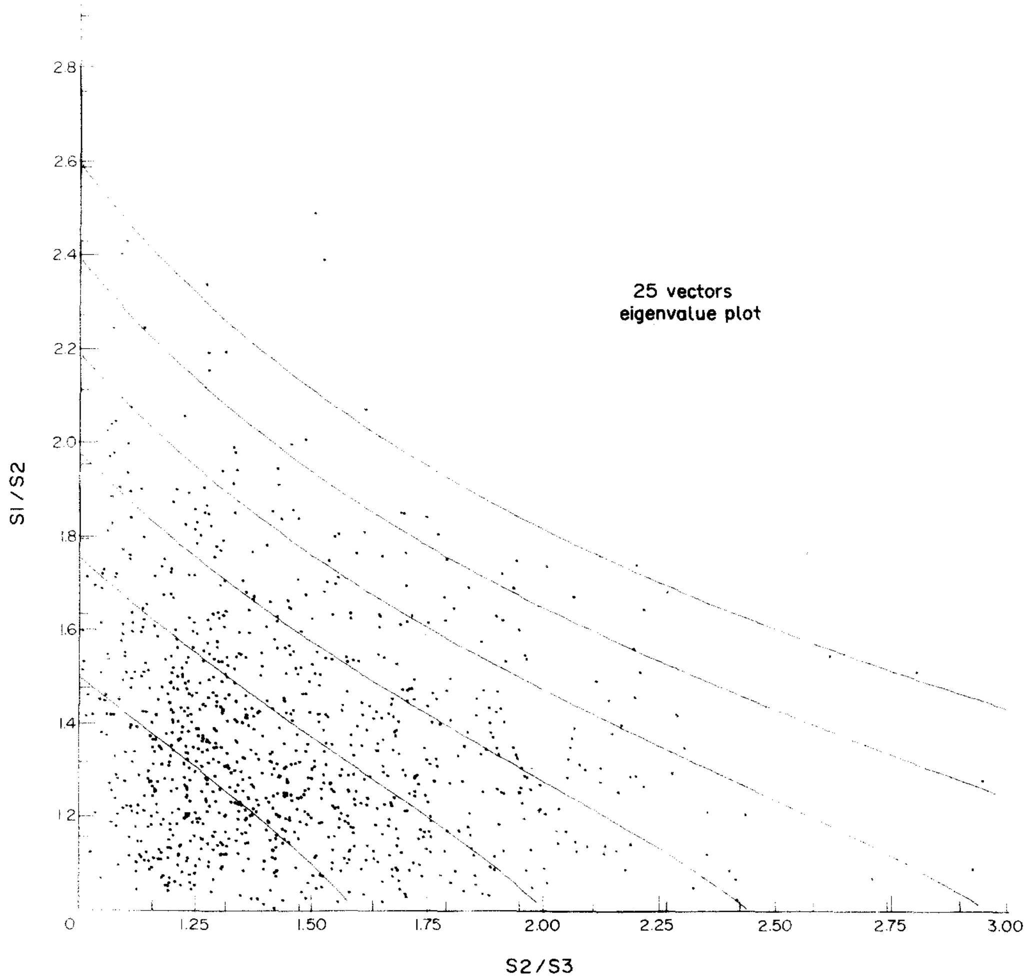

Fig. 2. Eigenvalue ratio plot showing the same data as Fig. 1. The distribution of eigenvalue ratios for random samples is compared to lines of constant I. The curves correspond to I = 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6.

this choice of strength parameter and suggests an alternative one.

DISADVANTAGES OF THE S_1/S_3 TEST

The most useful intensity parameter will be one which fulfils two purposes; to express the degree of preferred orientation shown by directions and to allow testing of the statistical significance of weak fabrics.

To give S_1/S_3 this second capability, Woodcock & Naylor (1983) determine, by Monte-Carlo methods, the distribution of this quotient for random samples from a uniform distribution. This allows S_1/S_3 to be used as a test statistic for uniformity. This Monte-Carlo simulation has been repeated by the present author to test the consistency of the results. The results based on a sample size of 25 are shown on the eigenvalue plot (Fig. 1) and are comparable with those of Woodcock & Naylor (1983 figs. 6 and 7) where the sample size was 50. An important feature is the asymmetry of the plotted eigenvalue ratios with respect to the K = 1 line on the plot. This asymmetry, noted also by Woodcock & Naylor (1983) but discounted as unimportant, implies that S_1/S_3 is distributed differently for girdle-type fabrics than for unimodal clusters. This asymmetry of the plotted random samples on the eigenvalue plot suggests that S_1/S_3 may not be the most efficient measure of deviation from uniformity.

A second disadvantage of using S_1/S_3 as a test-statistic for uniformity is that its distribution is not a simple function of sample size. This makes for inconvenience as tables or graphs similar to those of Woodcock & Naylor (1983 table 1 and fig. 9) have to be consulted when critical values of S_1/S_3 are sought.

AN ALTERNATIVE MEASURE OF FABRIC STRENGTH

The disadvantages mentioned above are not encountered if we choose an alternative parameter for fabric strength. We define Intensity (I),

$$I = \frac{15}{2} \sum_{i=1}^{3} \left(S_i - \frac{1}{3}\right)^2,$$

where S_i are the normalized eigenvalues of the orientation tensor. I varies from 0 to 5 for linear fabrics (unimodal clusters) and from 0 to 3.75 for planar fabrics (girdles). Fabrics which are geometrically similar will

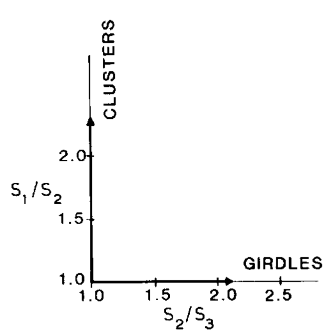

Fig. 3. The types of parent distribution used for the determination of the power of Woodcock and Mardia tests. The distributions are of the Dimroth–Watson type with concentration parameters κ ranging from -2.0 to 2.0 (shown by thick line).

yield identical values of I regardless of differences in sample size.

One of the benefits of defining I in this way is that it becomes simply related to the uniformity statistic S_u proposed by Mardia (1972, p. 276)

$$S_u = \frac{15n}{2} \sum_{i=1}^{3} \left(S_i - \frac{1}{3}\right)^2,$$

where n is the sample size. Consequently to convert the strength parameter to a uniformity statistic we use the equation

$$S_u = In.$$

Mardia's test is designed to test for uniformity against the alternative hypothesis of a Bingham distribution.

Unlike S_1/S_3, significance levels for S_u are independent of n, S_u having 95% and 99% values of 11.07 and 15.09, respectively (Mardia 1972, p. 277). Values of S_u exceeding these will suggest a deviation from uniformity. Data yielding S_u values which are too low on the other hand will also be suspect on the grounds of being too uniform for a random sample. The 5% and 1% values of S_u are 1.15 and 0.55, respectively.

That I is a more appropriate measure of deviation from uniformity can be appreciated by comparing the shape of the lines of constant I with the distribution of eigenvalue ratios from the random trials (Fig. 2). The constant I curves are given by the equation

$$I = \frac{15}{2}\left(\frac{a^2b^2 + b^2 + 1}{(ab + b + 1)^2} - \frac{1}{3}\right),$$

where a and b are the eigenvalue ratios S_1/S_2 and S_2/S_3, respectively. Comparison of Fig. 2 with Fig. 1 shows that the I curves parallel the density contours of points on this graph more closely than the lines of constant S_1/S_3 do. This means that I is less biased towards certain fabric shapes than is the parameter S_1/S_3.

COMPARISON OF THE POWER OF THE TESTS

Both tests are designed in such a way that random samples from a uniform distribution will yield, with

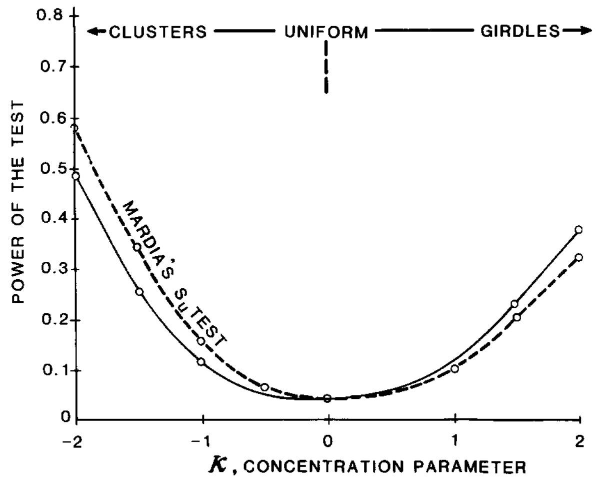

Fig. 4. A comparison of the power of Woodcock and Mardia tests when the alternative hypothesis is a Dimroth–Watson distribution. Each point is based on 1000 samples of 20 vectors.

known probability, values which exceed the critical values for the test. The power of such a test however is the ability of the test to reject uniformity when the random samples come from a parent distribution which is not a uniform distribution.

To calculate the power of each test, random samples were taken from a Dimroth–Watson distribution which is a particular case of the Bingham distribution, the distribution on which the Mardia test is based (see Mardia 1972, pp. 233–4). By varying the concentration parameter κ of the Dimroth–Watson distribution, simple clusters and simple girdles were considered (Fig. 3). The success rate with which each test rejected (correctly) the hypothesis of uniformity is expressed as a probability and labelled as the power in Fig. 4. Although the S_1/S_3 test is more powerful with some girdle-type distributions, the Mardia S_u test, as can be expected from a parametric test, is the more powerful test overall.

Although it is possible that the S_1/S_3 test might be more powerful in situations where the alternative hypothesis is a distribution other than a Bingham distribution, these situations still have to be defined.

Acknowledgements—I am grateful to Geoff Lloyd and Nigel Woodcock for their constructive comments on the manuscript.

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